



Line Flow Based WLS State Estimation Technique with Bad Data Measurements using PSO Algorithm

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ABSTRACT: State estimation techniques are commonly used by all the utilities to make the best possible estimate of the current system state from the existing set of redundant measurements. Typically, conventional estimates produce bus voltage magnitudes and angles which need to be converted later into line loadings to conduct security analysis. Here, a novel line flow based state estimation technique that provides the output in terms of real and reactive power flows, and bus voltage magnitudes has been developed and resolved using PSO algorithm. When applied to standard test systems in the presence of various percentages of bad measurements, the suggested method has been observed to provide better results in terms of standardized error values and net computation time than conventional WLS technique.

KEYWORDS: State Estimation, Weighted Least Squares method, Line flowbased WLS, PSO and Power System.

I.INTRODUCTION

State estimation techniques are basically data processing algorithms applied to power systems to obtain the best estimate of current operating state from the available set of redundant measurements and network topology data. SE problems are minimization problems and in accordance with the problem formulation any one of the standard minimization techniques is applied for problem solving. An L norm based SE method devised using the Newton's technique is set out in (1). Although the approach is reliable, resulting in relatively good results even for high noise component measurements, L problems are difficult to calculate and time consuming. In view of rotation based SE methodology in which the least squares problem based on equality constraints is decomposed into two unconstrained least squares problems with the goal of minimizing the impact of assigning more weights to equality constraints is explained in (2). The proposed method is numerically robust but has low speed characteristics. A simple decoupled SE technique based on equal current injections and rectangular coordinates is discussed in (3). In addition, this is a break through technique as it resulted in similar sub-gain matrices that had to be modified and factored only once. This method, which is impressive from the point of view of speed and applicability, tends to generate a compromising calculation by retaining the same weighting factors for both real and reactive measurements. A sparse block QR decomposition based on the Given transformation for SE algorithm was suggested in (4). This technique is powerful which makes it suitable for ill conditioned power systems but triangular factorization of the coefficient matrix may introduce numerical round off problems. Singular Value Decomposition technique that is relevant even if a part of the network is unobservable is presented in (5). Though it is suitable for ill conditioned systems, partitioning of the measurement matrix involves an additional matrix multiplication which is not commonly desired. The line flow and bus voltage magnitude load flow model described in (6) has been used in this paper to develop a similar SE model that was overcome using WLS technique by using the PSO algorithm. This method tends to avoid most of the factorization related matrix manipulation problems stated so far. The presence of bad data has a important impact over the quality of the estimate generated by the least squares based estimator and hence special techniques were needed to detect them and quantify their effects. A linear recursive bad data identification technique based on power system decomposition has been presented in (7). Neural network based filter has been applied for bad data detection and identification in (8), in which once trained, the filter easily detects most of the measurement errors simultaneously by comparing the square difference of raw measurements and their corresponding estimated values with some certain thresholds. Bad data prefiltering using wavelet transform has been presented in (9) and this approach identifies and filters out the bad data



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even before the state estimation algorithm estimates the system state. An identification algorithm based on the largest normalized residual considering statistical correlation among the measurements is presented in (10). As the proposed method here uses a constant Jacobian, unlike the conventional WLS estimator, the effect of bad measurements over the estimate has been significantly reduced and subsequently it doesn't need a separate algorithm to filter out the bad measurements. Soft computing algorithms have been played a major role in solving optimization problems over the last few decades. Evolutionary programming algorithms are impressive in terms of their ability to evade local maxima and minima. Out of the many evolutionary algorithms PSO has been broadly used from the point of view of assured convergence and programming flexibility. PSO algorithm has been efficiently implemented for solving the problem of SE inspite of the apprehensions such as larger computational time etc (11-13). In this work a line flow based WLS state estimation problem has been developed and it has been solved by PSO technique in the absence as well as the presence of bad measurements for various standardized IEEE test systems.

II. CONVENTIONAL WLS STATE ESTIMATION

The aim of state estimation technique is to find out a set of state vectors that minimize the measurement residuals. The problem is basically a minimization problem and since the quantities involved are nonlinear, it is a problem of minimizing a nonlinear objective function that can be conveniently achieved using least squares technique. The measurements are expressed as a function of the state vector of the system as

$$Z = h(x) + V \quad (1)$$

Here, the measurement errors which normally show normal distribution around a zero mean are assumed to be independent of each other. Each one of the measurement is assigned a acceptable weightage reflecting the accuracy and the reliability of that particular measurement whose values are decided based upon several factors such as the condition of the measuring equipment, noise of the telemetry channel etc.

The objective of WLS state estimator is to generate a suitable set of state variables in terms of bus voltage magnitudes and angles to minimize the weighted sum of the squares of the measurement errors, to achieve this the objective function is formulated as

$$J(x) = [z - h(x)]^T W [z - h(x)]$$

where W is the weightage matrix, which is a diagonal matrix formed by measurement covariances when the measurements are independent of each other. The above equation is solved iteratively for estimating the state vector that minimizes J . At the end of every K^{th} iteration, the state vector is updated using the correction vector as

$$x_{k+1} = x_k + \Delta x \quad (3)$$

in which Δx obtained by solving the equation

$$\Delta x_k = [H^T W H]^{-1} H^T W [z - h(x)] \quad (4)$$

Where H stands for the Jacobian and $[H^T W H]$ represents the gain matrix.

III. PROPOSED METHOD

The proposed method tries to solve the SE problem by applying WLS technique on a line flow based model (6) which is constructed using power balance equations, line voltage equations and loop phase angle equations.

The general power balance equations for the system are written as

$$P(T) = e^{-\frac{\Delta F}{T_i}} \quad (5)$$

$$A.p - P_{GL} - A'.l = 0 \quad (6)$$

$$A.q - Q_{GL} - A'.m - H.V^2 = 0 \quad (7)$$



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Where A and A' are defined as bus incidence and modified bus incidence matrices in which all +1's in A are set to zeros. l and m represent the real and reactive power losses in the transmission lines, H is an $(n-1)$ diagonal matrix formed by the sum of charging and compensating susceptances at each bus bar, P_{GL} and Q_{GL} are the real and reactive bus power injections, P and Q are the real and reactive power flows measured at the receiving end of the transmission line. When the reactive mismatch equations are deleted at generator buses then the above equation can be rewritten as

$$A_1 \cdot q - Q_{GL} l - A_1 \cdot m - H_1 \cdot V^2 = 0 \quad (8)$$

in which H' is a diagonal matrix with most effective elements corresponding to load buses present in it. The line voltage equations are composed in view of a network branch model developed without taking and considering the shunt elements including the line capacitances as,

$$2Rp + 2Xq - (\Lambda A_{1+}^T + A_{1-}^T) V^2 = -k + \Lambda A_C^T V_{PV}^2 \quad (9)$$

Where k is the vector of apparent line losses, A_C is the bus bar incidence matrix corresponding to only the PV buses, V_{PV}^2 is the vector of the square of the generator bus voltage, Λ is the diagonal matrix of order 1 with the values equal to the square of the tap settings, A_{1+} and A_{1-} are the positive and negative element parts of A_1 , R and X are the diagonal resistance and reactance matrices.

The loop phase angle equations are written based on the fact that the algebraic sum of phase angle drops around independent loops are zeros.

$$CXp - CRq = 0 \quad (10)$$

The real and reactive bus powers as a function of real line flows, reactive line flows, real line loss and reactive line loss and V_m^2 can be written as

$$P_i = \sum_{j=1}^{n_l} A_{ij} P_j - \sum_{j=1}^{n_l} A'_{ij} l_j \quad (11)$$

$$Q_i = \sum_{j=1}^{n_l} A_{ij} Q_j - \sum_{j=1}^{n_l} A'_{ij} m_j + H_{ii} V_i^2 \quad (12)$$

Treating P , Q and V_m as state variables $[X]$ the measurement set Z can be represented as

$$[Z] = [f(x)] \quad (13)$$

where $[Z] = [P \ Q \ p \ q \ V^2]^T$. The WLS objective function is written as

$$\text{Min } \varphi = [f(x) - Z]^T [w] [f(x) - Z] \quad (14)$$

As the above equation does not include line capacitances and shunt susceptances, it is inadequate to estimate the system state. However the problem is made solvable by considering the constraint equations involving branch voltage drop and phase angle variations. These equations are written as

$$h(x) = 2Rp + 2Xq - (\Lambda A_{1+}^T + A_{1-}^T) V^2 = 0 \quad (15)$$

$$g(x) = CXp - CRq - C\alpha = 0 \quad (16)$$

The constrained optimization problem involving equations (14), (15) and (16) is converted into an unconstrained problem through Lagrangian multipliers λ and μ as

$$\text{Min } \varphi = [f(x) - Z]^T [W] [f(x) - Z] - \lambda h(x) - \mu g(x) \quad (17)$$

Linearising the above equation around a known operating point x^0 and then differentiating it and equating it to zero will result in a matrix equation of the following form



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$$\begin{bmatrix} 2F^T W F & -H^T & -G^T \\ H & 0 & 0 \\ G & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta x \\ \lambda \\ \mu \end{bmatrix} = \begin{bmatrix} -2F^T W (f(x^0) - Z) \\ -h(x^0) \\ -g(x^0) \end{bmatrix} \quad (18)$$

where F and G represent the jacobian matrices. These matrices are constant ones and the right hand side vector is split into two groups, one consisting of the bus power injections and generator bus voltages and the other consisting of nonlinear loss term and charging and compensating powers. So the vector right hand side is partially linearised. The algorithm for solving the objective function given in (18) is explained in the section below.

IV. INTRODUCTION OF PSO

PSO was first introduced in 1995 by Kennedy and Eberhart and is a heuristic optimization technique caused by swarm intelligences of animals such as bird flocking, fish schooling. A swarm of particles is the solution to the optimization problem. Each particle adjusts its position based on its own experience and the experience of its neighboring particles. The position and velocity of i^{th} particle in an N – dimensional search space is expressed as

$$X_i = (x_{i1}, x_{i2}, \dots, x_{in})$$

$$v_i = (v_{i1}, v_{i2}, \dots, v_{in})$$

The best position achieved by a particle is recorded and is denoted by

$$P_{best\ i} = (x_{i1}^{P_{best}}, \dots, x_{in}^{P_{best}})$$

The best particle among all the particles in the population is represented by

$$G_{best\ i} = (x_{i1}^{G_{best}}, \dots, x_{in}^{G_{best}})$$

The updated velocity and position of each particle in $(k + 1)^{th}$ step are calculated as follows

$$X_i^{k+1} = X_i^k + V_i^{k+1} \quad (19)$$

where,

$$V_i^{k+1} = wV_i^k + C_1 rand_1 (x_i P_{best\ i}^k - x_i^k) + C_2 rand_2 (G P_{best\ i}^k - x_i^k) \quad (20)$$

where

$$C1 = C_1 \max - \left\{ \frac{C_{1\max} - C_{1\min}}{Iter_{\max}} \right\} x Iter \quad (21)$$

$$C2 = C_2 \max - \left\{ \frac{C_{2\max} - C_{2\min}}{Iter_{\max}} \right\} x Iter \quad (22)$$

$$w = w_{\max} - \left\{ \frac{w_{\max} - w_{\min}}{Iter_{\max}} \right\} x Iter \quad (23)$$

In this velocity updating process, the acceleration coefficients C_1 , C_2 and weight parameter 'w' are predefined and $rand_1$ and $rand_2$ are uniformly generated random numbers in the range of [0, 1] and this velocity update is done until stop requirement is reached/met.

V. PSO ALGORITHM

1. Choose the population size, the number of generations, W_{\min} , W_{\max} , $C_{1\min}$, $C_{1\max}$, $C_{2\min}$, $C_{2\max}$, p_{best} , g_{best} .
2. Randomly configure all particles' velocity and position, ensuring that they are within limits. The entities here represent the real and reactive power flows and bus voltage magnitudes.
3. Set the generation counter $t=1$.
4. Use equation (17) to evaluate the fitness of each particle according to the objective function.



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5. Compare the particle's fitness function with its $P_{best i}$. If the current value is better than $P_{best i}$, then set $P_{best i}$ is equal to the current value. Identify and allocate the particle to Gbest with the best success in the neighborhood so far.
6. Update velocity by using the global best and individual best of the particle.
7. Update position by using the updated velocities. Each particle will change its position.
8. If the stopping criteria is not fulfilled set $t=t+1$ and go to step 4. If not quit.

VI. SIMULATION AND RESULTS

The proposed LFBSE problem has been solved using PSO technique by selecting a population size of 20 and generation size of 50 and it has been tested on standard IEEE 14, 30 and 57 bus test systems. By adding a small percentage of noise to the values obtained from the Newton Raphson load flow the measurement vector has been generated. Bus voltage magnitudes at the load buses and real and reactive power flows through the lines were taken as state variables. All the line flows, bus power injections and bus voltage magnitudes at the even numbered buses were considered in the measurement set to achieve necessary redundancy. In order to test the algorithm's output in the presence as well as the absence of bad measurements, 5, 10 and 15 numbers of bad measurements are inserted randomly in each of the measurement array. The performance of the algorithm has been confirmed by comparing the results of the proposed method against the results obtained using standard WLS state estimation and LFWLS State Estimation. With a flat start and a convergence tolerance of 0.0001, the algorithms were tested. Three performance indices are defined to validate the performance of the proposed technique. They are ΔV_{rms} , ΔP_{rms} and ΔQ_{rms}

$$\Delta V_{rms} = \sqrt{\frac{1}{nb} \sum_i^{nb} (V_i^t - V_i)^2} \quad (24)$$

$$\Delta p_{rms} = \sqrt{\frac{1}{nl} \sum_i^{nl} (P_i^t - P_i)^2} \quad (25)$$

$$\Delta q_{rms} = \sqrt{\frac{1}{nl} \sum_i^{nl} (q_i^t - q_i)^2} \quad (26)$$

Tables 1, 2 and 3 compare the performance of the proposed method with WLS and LFWLS estimation algorithm in terms of the performance indices defined in 24, 25 and 26 and NET. The performance of the algorithm is also demonstrated in Fig 1 to 12 by bar charts.

Measurements	Method	NET in ms	ΔV_{rms}	ΔP_{rms}	ΔQ_{rms}
0	WLS	197	0.148	0.1351	0.1643
	LFWLS	123	0.093	0.110	0.111
	LFWLS-PSO	129	0.0918	0.1097	0.1097
5	WLS	198	0.1479	0.1286	0.1631
	LFWLS	123	0.0928	0.1074	0.1094
	LFWLS-PSO	130	0.0913	0.1045	0.1085
10	WLS	198	0.1435	0.1277	0.1573
	LFWLS	124	0.0669	0.1034	0.1083
	LFWLS-PSO	130	0.0616	0.1033	0.1079
15	WLS	198	0.1421	0.1215	0.138
	LFWLS	123	0.0259	0.1027	0.1078
	LFWLS-PSO	131	0.0245	0.1026	0.107

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Fig.1 Measurement Vs ΔV_{rms} (14 Bus)

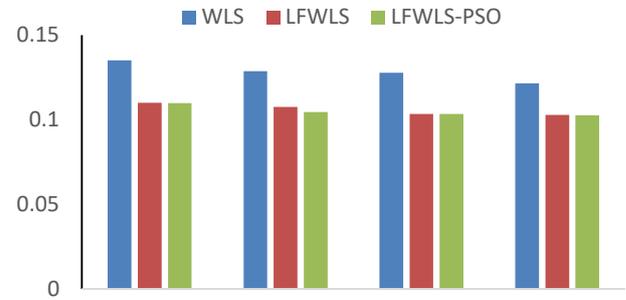


Fig.2 Measurement Vs ΔP_{rms} (14 Bus)

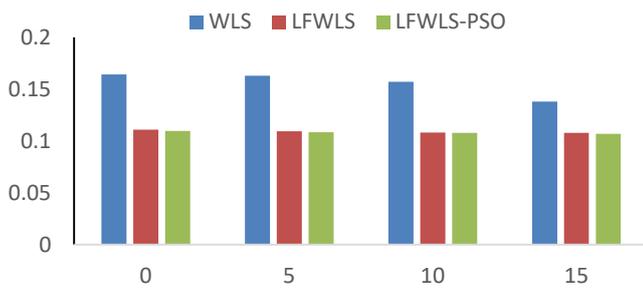


Fig.3 Measurement Vs ΔQ_{rms} (14 Bus)

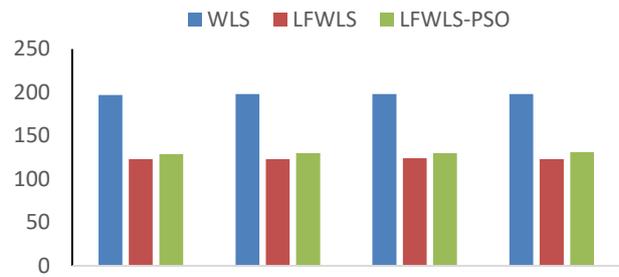


Fig.4 Measurement Vs NET (14 Bus)

Measurements	Method	NET in ms	ΔV_{rms}	ΔP_{rms}	ΔQ_{rms}
0	WLS	450	0.0984	0.3824	0.2117
	LFWLS	167	0.0795	0.2173	0.1325
	LFWLS-PSO	176	0.0694	0.2165	0.1318
5	WLS	452	0.0877	0.3794	0.2109
	LFWLS	166	0.0418	0.2159	0.1319
	LFWLS-PSO	177	0.0331	0.2157	0.1314
10	WLS	450	0.0641	0.3756	0.2099
	LFWLS	166	0.0345	0.2138	0.1311
	LFWLS-PSO	176	0.0329	0.213	0.1307
15	WLS	450	0.0478	0.3743	0.2081
	LFWLS	166	0.0298	0.213	0.1305
	LFWLS-PSO	176	0.0257	0.2024	0.1299

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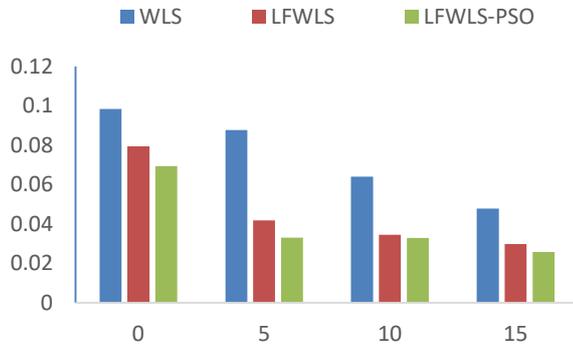


Fig.5 Measurement Vs ΔV_{rms} (30 Bus)

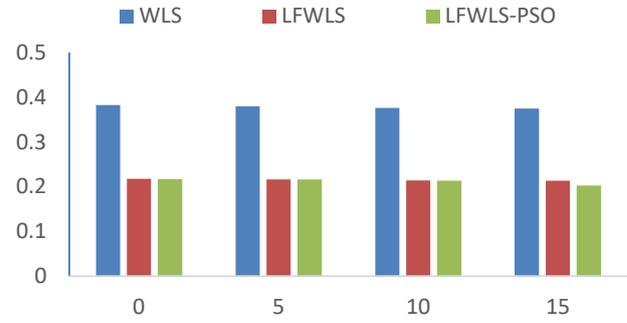


Fig. 6 Measurement Vs ΔP_{rms} (30 Bus)

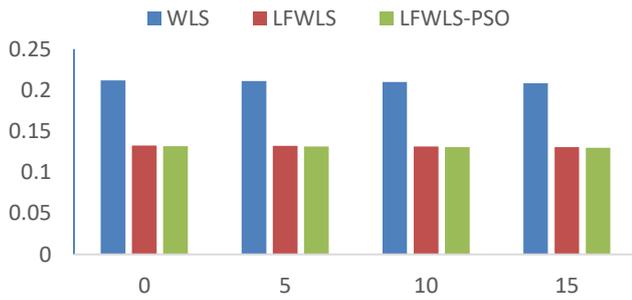


Fig.7 Measurement Vs ΔQ_{rms} (30 Bus)

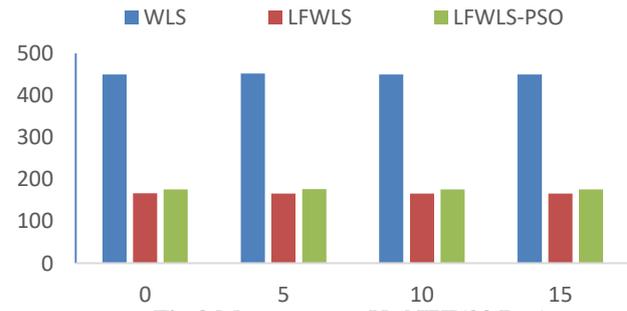


Fig.8 Measurement Vs NET(30 Bus)

Measurements	Method	NET in ms	ΔV_{rms}	ΔP_{rms}	ΔQ_{rms}
0	WLS	579	0.0833	0.2579	0.1346
	LFWLS	207	0.0304	0.1173	0.1091
	LFWLS-PSO	218	0.0303	0.1161	0.1083
5	WLS	583	0.0829	0.2553	0.1332
	LFWLS	207	0.0298	0.1164	0.1083
	LFWLS-PSO	218	0.0284	0.1152	0.1305
10	WLS	580	0.0823	0.2527	0.132
	LFWLS	208	0.0286	0.1158	0.1071
	LFWLS-PSO	219	0.0272	0.1147	0.1293
15	WLS	581	0.0815	0.2502	0.1313
	LFWLS	208	0.0272	0.115	0.106
	LFWLS-PSO	220	0.0264	0.114	0.1288

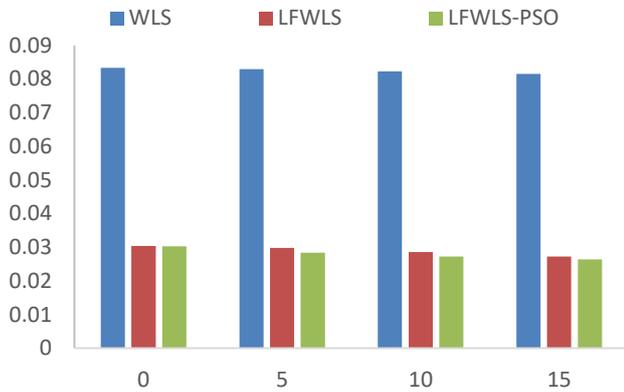


Fig.9 Measurement Vs ΔV_{rms} (57 Bus)

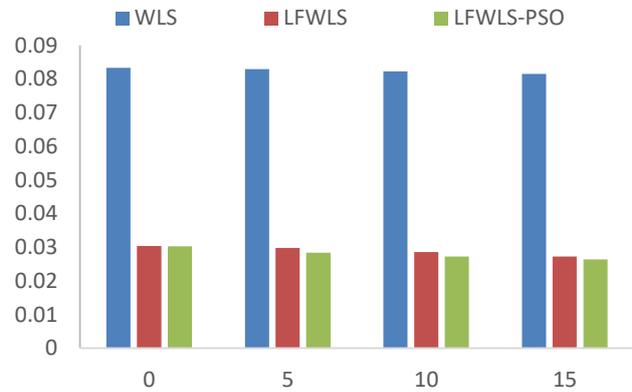


Fig.10 Measurement Vs ΔP_{rms} (57 Bus)

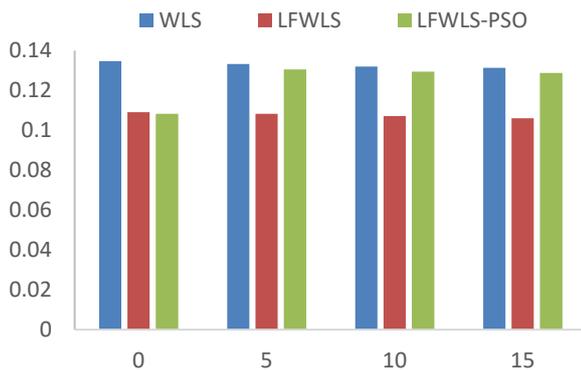


Fig.11 Measurement Vs ΔQ_{rms} (57 Bus)

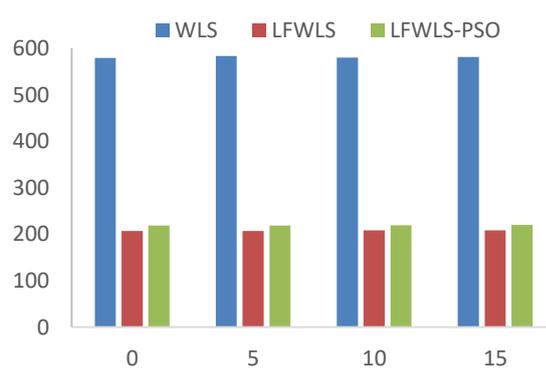


Fig.12 Measurement Vs NET(57 Bus)

VII. CONCLUSION

In this paper, a new state estimation technique that results in the formation of constant jacobian matrix has been introduced and solved for various percentages of bad measurements by PSO technique. The results indicate that when solved using PSO, the normalized value of the error between the actual values and estimated values of the state variables is considerably lower than that of the conventional WLS technique in case of proposed method. Furthermore, it can be observed that the presence of bad measurements has a significant impact on the accuracy of estimation in the conventional WLS technique whereas the proposed LFBSE technique does not. In conventional WLS estimated state variables deviates more from their true values and this deviation increases with the increase in the number of bad measurements. But this deviation is slightly less in the proposed method because that the jacobian turns out to be a constant matrix. PSO has marginally increased the computation time and this increased computational time could be compromised against by the reduction of the normalized error values. Hence it can be concluded that the proposed PSO based LFWLS generates more accurate estimates than the conventional WLS method and it takes lesser computation time and shows less sensitivity to the presence of bad measurements, making it suitable for real time studies.



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NOMECLATURE

<i>LFBSE</i>	<i>Line Flow Based State Estimation</i>
<i>SE</i>	<i>State Estimation</i>
<i>WLS</i>	<i>Weighted Least Squares</i>
<i>PM</i>	<i>Proposed Method</i>
<i>PSO</i>	<i>Particle Swarm Optimization</i>
<i>LFWLS</i>	<i>Line flow based WLS</i>
<i>Z</i>	<i>Measurement Vector</i>
x^o	<i>Initially assumed values of state vector</i>
X_k	<i>State vector at K^{th} iteration</i>
X_{k+1}	<i>State vector at $K + 1^{th}$ iteration</i>
ΔX_k	<i>State correction vector after K^{th} iteration</i>
<i>H</i>	<i>Jacobian Matrix</i>
<i>h(x)</i>	<i>Measurement function</i>
<i>J(x)</i>	<i>Objective function</i>
<i>V</i>	<i>Vector of measurement residues</i>
H^TWH	<i>Gain matrix</i>
<i>P_i</i>	<i>Real bus power injection</i>
<i>Q_i</i>	<i>Reactive bus power injection</i>
<i>A_{ij}</i>	<i>ijth element of bus incidence matrix</i>
<i>A'_{ij}</i>	<i>ijth element of modified bus incidence matrix</i>
<i>P_j</i>	<i>Real power flow in jth line</i>
<i>l_j</i>	<i>Real power loss in jth line</i>



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Q_j	Reactive power flow in j^{th} line
m_j	Reactive power loss in j^{th} line
H	Diagonal matrix formed by the sum of shunt and compensating susceptances at each bus
X	Diagonal matrix of line reactances
A	Diagonal matrix of order 1 with the values equal to the square of the tap settings
A_{1+} and A_{1-}	Positive and negative element of A_1
λ and μ	Lagranjian Multipliers
C	Loop incidence matrix
α	Phase angle of the phase shifter, taken as 1 otherwise
$\Delta V_{rms}, \Delta p_{rms}, \Delta q_{rms}$	$\Delta V_{rms}, \Delta p_{rms}, \Delta q_{rms}$ –Root Mean Square values of the corresponding quantities
V_i^t, P_i^t and Q_i^t	True values of the respective quantities on i^{th} bus
X_i^k	Position of individual i at iteration k
X_{i+1}^k	Position of individual i at iteration $k + 1$
k	Velocity of individual i at iteration k
w	weight parameter
C_1	Cognitive factor
C_2	Social factor
P_{best}^k	best position of individual i until iteration k
G_{best}^k	best position of group until iteration k
$rand_1, rand_2$	random numbers between 0 and 1
w_{min}, w_{max}	initial and final weights
C_{1min}, C_{1max}	initial ad final cognitive factors
C_{2min}, C_{2max}	initial and final social factors
$Iter_{max}$	maximum iteration number
$Iter$	current iteration number
$rand_3$	random numbers between 0 and 1